STRUCTURE OF A COLLISIONLESS BOUNDARY LAYER AND TURBULENT DAMPING OF IONS

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The flow of a low-pressure plasma in a MHD channel is unstable in a number of cases. The instability can be caused by a current flowing across the magnetic field. In this study we investigate an unstable, turbulent flow of a rarefied plasma near the "magnetized electrodes," representing plane magnetic dipoles. Owing to the growth of microscopic turbulence near the electrodes, the maximum density of the current that is induced in the plasma is localized and turbulent damping of the incoming flow occurs. The energy of damping goes into the turbulent heating of the plasma. Under these conditions a structure of the boundary layer is found for a stationary flow. The characteristic transverse dimension of the boundary layer is considerably less than the particle mean free path; therefore, such a boundary layer can be called "collisionless."

In a number of works [1-3] in the investigation of the laminar flow of a rarefied low-pressure plasma in a MHD channel it was shown that near the electrodes, "nondissipative" boundary layers can form, the dimension of which, in order of magnitude, equal the Larmor radius or Debye radius of electrons.

In such boundary layers, the plasma is collisionless and energy is not extracted.

In the boundary layer the density of the current flowing across the magnetic field can be considerably greater than the critical density ($j \gg j^*$) [4], beginning at which the plasma becomes unstable. Under these conditions a special role should be played by collective processes, leading to "collisionless dissipation" of energy, turbulent heating and damping of particles, and the formation of a collisionless structure of the boundary layer.

We theoretically justify the possibility of the formation of collisionless boundary layers in the flow of a rarefied plasma, having characteristic dimension much less than the particle mean free path, in order of magnitude, equal to c/ω_{pi}, where $\omega_{pi} = (4\pi ne^2/M)^{1/2}$ is the ion-plasma frequency.

1. We consider the plane flow of a rarefied low-pressure plasma ($\beta = 4\pi nT_e/H^2 \ll 1$) near an electrode (Fig. 1). We assume that the characteristic frequencies of all the motions are much less than the electron Larmor frequency $\omega_{H_e} = eH/mc$, and the plasma is quasineutral ($n_e = n_i = n$). Under these conditions, the equations for the electron and ion components of the plasma take the form

$$\begin{gathered}
\frac{\partial n}{\partial t} + \operatorname{div}(n\mathbf{v}_e) = 0, \quad \frac{\partial n}{\partial t} + \operatorname{div}(n\mathbf{v}_i) = 0 \\
mn\left(\frac{\partial \mathbf{v}_e}{\partial t} + (\mathbf{v}_e\nabla)\mathbf{v}_e\right) = -en\mathbf{E} - \nabla p_e - \frac{en}{c}[\mathbf{v}_e\mathbf{H}] - \mathbf{R}_f \\
Mn\left(\frac{\partial \mathbf{v}_i}{\partial t} + (\mathbf{v}_i\nabla)\mathbf{v}_i\right) = en\mathbf{E} + \frac{en}{c}[\mathbf{v}_i\mathbf{H}] + \mathbf{R}_f \\
\operatorname{rot}\mathbf{H} = \frac{4\pi ne}{c}(\mathbf{v}_i - \mathbf{v}_e), \quad \frac{1}{c}\frac{\partial \mathbf{H}}{\partial t} + \operatorname{rot}\mathbf{E} = 0
\end{gathered} \tag{1.1}$$

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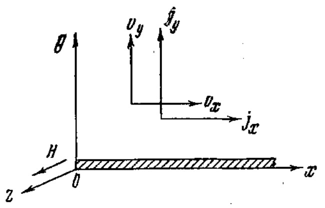

Fig. 1

where $\mathbf{v}_e$ and $\mathbf{v}_i$ are the mean velocities of electrons and ions; m and M are their respective masses; $p_e = nT_e$ is the gas-kinetic pressure of the electron (we assume that $T_e \gg T_i$); T_e and T_i are the temperatures of electrons and ions, respectively; $\mathbf{R}_f$ is the effective force of friction, due either to the "infrequent" Coulomb collisions or to the collective collisions. Introducing the mean-mass velocity

$$\mathbf{v} = (m\mathbf{v}_e + M\mathbf{v}_i) / M_0, \quad M_0 = m + M$$

and eliminating the electric field E in (1.1), we obtain the system of equations

$$\begin{gathered}
\frac{\partial n}{\partial t} + \operatorname{div}(n\mathbf{v}) = 0 \\
M_0 n\left(\frac{\partial \mathbf{v}}{\partial t} + (\mathbf{v}\nabla)\mathbf{v}\right) = \frac{1}{4\pi}[\operatorname{rot}\mathbf{H}, \mathbf{H}] - \nabla p_e - \frac{mMc^2}{M_0(4\pi e)^2}(\operatorname{rot}\mathbf{H}\cdot\nabla)\left(\frac{\operatorname{rot}\mathbf{H}}{n}\right) \\
\frac{\partial \mathbf{H}}{\partial t} = \operatorname{rot}[\mathbf{v}\mathbf{H}] + \operatorname{rot}\left(\frac{c\mathbf{R}_f}{ne}\right) - \frac{Mc}{4\pi e M_0}\operatorname{rot}\left[\frac{\operatorname{rot}\mathbf{H}}{n}, \mathbf{H}\right] + \left(\frac{Mc}{4\pi e M_0}\right)^2 \frac{mc}{e} \times \\
\times \operatorname{rot}\left\{\left(\frac{\operatorname{rot}\mathbf{H}}{n}\nabla\right)\frac{\operatorname{rot}\mathbf{H}}{n}\right\} - \frac{Mmc^2}{4\pi e^2 M_0}\operatorname{rot}\left\{\left(\frac{\partial}{\partial t} + (\mathbf{v}\nabla)\right)\frac{\operatorname{rot}\mathbf{H}}{n} + \left(\frac{\operatorname{rot}\mathbf{H}}{n}\nabla\right)\left(\mathbf{v} + \frac{mc}{4\pi e M_0}\frac{\operatorname{rot}\mathbf{H}}{n}\right)\right\}
\end{gathered} \tag{1.2}$$

The system of equations of two-fluid hydrodynamics in such form had been repeatedly used earlier, for example in [5]. In our study the initial system of equations (1.2), being complete, will be used for the solution of a particular problem – the investigation of the structure of an unstable, turbulent flow of a rarefied plasma in a boundary layer (Fig. 1).

With this aim we analyze below one of the possible mechanisms of collective friction, resulting from the growth of microscopic turbulence in the boundary layer, and as a supplement to the system (1.2) we give the closing equation for the energy density of the unstable oscillations.

2. The plasma flow in the boundary layer is unstable if the current velocity

$$|\mathbf{u}| = \frac{c}{4\pi e n}|\operatorname{rot}\mathbf{H}|$$

exceeds a certain critical value u* [4]:

$$|\mathbf{u}| = \frac{c}{4\pi e n}|\operatorname{rot}\mathbf{H}| > u^* \tag{2.1}$$

In a plasma having "hot" electrons ($T_e \gg T_i$) the most unstable oscillations are of "ion-acoustic" type, having, in agreement with [4], the following values for frequency ω_k and growth rate γ_k:

$$\begin{gathered}
\omega_k = \mathbf{k}\mathbf{v} + k v_s \alpha, \quad \alpha = (1 + k_\perp^2 \rho_{H_e}^2)^{-1/2} \\
\gamma_k = \left(\frac{\sqrt{\pi} k v_s \alpha^3}{2k_\parallel v_{T_e}}\right)\left(\frac{c\mathbf{k}\operatorname{rot}\mathbf{H}}{4\pi e n} - \omega_k\right), \quad k^2 = k_\perp^2 + k_\parallel^2
\end{gathered} \tag{2.2}$$

Here we have introduced the notation

$$v_s = \left(\frac{T_e}{M}\right)^{1/2}, \quad \rho_{H_e} = \frac{v_{T_e}}{\omega_{H_e}}, \quad v_{T_e} = \left(\frac{2T_e}{m}\right)^{1/2}, \quad \omega_{H_e} = \frac{eH}{mc}$$

where v_s is the ion-acoustic velocity, $k_\parallel$ and $k_\perp$ are projections of the wave vector **k** on the longitudinal and transverse directions of the magnetic field, respectively.

An investigation of an ion-acoustic instability, performed in [4], shows that the most unstable perturbations are "skew perturbations," for which $k_\parallel / k \approx (m/M)^{1/2}$ and the wavelength is of the order of the Larmor electron radius $k^{-1} \approx \rho_{He}$. The instability boundary lies at the level of ion-acoustic velocity $u^* \approx v_s$, and the maximum growth rate, in order of magnitude, is equal to the hybrid frequency $\omega_{e,i} = (\omega_{Hi}\omega_{He})^{1/2}$, where $\omega_{Hi} = eH/Mc$ is the Larmor ion frequency.

We assume that everywhere in the boundary layer the current velocity exceeds the critical value, and the plasma state is unstable. Growth of an instability leads to the occurrence of a collective frictional force R_f between the electron and ion components, which according to [4] equals

$$\mathbf{R}_f = \sum_{\mathbf{k}} \mathbf{k}\gamma_{\mathbf{k}} \frac{W_{\mathbf{k}}}{\omega_{\mathbf{k}}} = \sum_{\mathbf{k}} \left(\frac{\sqrt{\pi} k^2 v_s \alpha^3}{2\omega_k k_{\parallel} v_{T_e}}\right)\left(\frac{c \operatorname{rot} \mathbf{H}}{4\pi en} - v_s \frac{\mathbf{k}}{k}\right) W_{\mathbf{k}} \tag{2.3}$$

Here the summation is carried out over all wave numbers k for which the oscillations are unstable. The maximum contribution to the sum is made by the wave numbers for which the instability growth rate is a maximum; therefore, everywhere below, the summation will be replaced by the single term that has maximum instability of the components of the wave vector k^*:

$$k_{\parallel}{}^* \approx k^* (m / M)^{1/2}, \quad k^* \approx \rho_{H_e}{}^{-1}$$

In Eq. (2.3) W_k is the spectral density of the oscillation (noise energy). In quasilinear theory [6], which describes the weakly turbulent plasma state and on the basis of which we will consider below the principal macroscopic effects owing to the growth of ion-acoustic turbulence, the growth in oscillation energy is given by the following kinetic equation:

$$\left[\frac{\partial}{\partial t} + \left(\frac{\partial \omega_k}{\partial \mathbf{k}} \nabla\right)\right] \ln W_k = 2\gamma_k = \left(\frac{\sqrt{\pi} k v_s \alpha^3}{k_{\parallel} v_{T_e}}\right)\left(\frac{c\mathbf{k} \operatorname{rot} \mathbf{H}}{4\pi en} - \omega_k\right) \tag{2.4}$$

Here $\partial\omega_k/\partial k$ is the oscillation group velocity, for an ion-acoustic instability, equal to the mean-mass velocity v.

Thus, the system of equations (1.2) together with (2.3) and (2.4) is self-consistent and qualitatively correctly indicates the principal macroscopic effects that result from the growth of microscopic ion-acoustic turbulence.

In addition to the ion-acoustic instability, which has a low excitation threshold $u^* \approx v_s$ and which gives a maximum contribution to the anomalous plasma resistance for current velocities significantly greater than the ion-acoustic velocity $u \gg v_s$, microscopic turbulent oscillations can be excited in the electron fluid, leading to the appearance of a characteristic macroscopic effect of "anomalous-electronic-viscosity" type. The physical sense of this effect can be understood by converting to the equation of motion of electrons in system (1.1).

We first consider an instability of turbulent oscillations (rot $E \neq 0$) in an incompressible electron fluid (div $v_e = 0$). For definiteness we consider plane flow of the electron fluid. The mean electron velocity is directed along the y axis and is inhomogeneous along the x axis; the magnetic field is directed along the z axis.

We introduce the following notation for velocity v_e and vorticity Ω:

$$\mathbf{v}_e = -\frac{c}{4\pi en} \operatorname{rot} \mathbf{H}, \quad \Omega = \operatorname{rot} \mathbf{v}_e = \frac{c}{4\pi en} \Delta \mathbf{H}$$

Applying the rot (= curl) operation to the electron equation of motion and performing a Fourier transformation for small perturbations that can be represented in the form $\sim H_{\sim}(x) \exp(-i\omega t + iky)$, we obtain a dispersion equation which describes the flow instabilities of an incompressible electron fluid:

$$H_{\sim}''\left(-\omega a^2 - ia^2 \nu_f + \frac{e}{mc} a^4 k H_0'\right) + H_{\sim}\left[(1 + k^2 a^2) - \frac{k^3 e a^4 H_0'}{mc} - \frac{e}{mc} a^4 k H_0''' + ik^2 a^2 \nu_f\right] = 0 \tag{2.5}$$

Here $\nu_f = \partial R_f/\partial v_e mn$ is the effective frequency of elastic scattering of electrons, $a = c/\omega_{pe}$ is the dispersion scale, $H_{\sim}(x)$ is the oscillation amplitude of the magnetic field, which depends on the x coordinate. Equation (2.5) has the same structure as the equations of turbulent oscillations of an ideal fluid.

We consider the spectrum of the unstable oscillations in the quasiclassical approximation $kd \ln H/dx \gg 1$). Applying to Eq. (2.5) the method of "quasiclassical quantization," for the limiting case when $kd \ln H/dx \gg 1$, we obtain the following values for the frequency ω_k and the growth rate γ_k of the unstable oscillations:

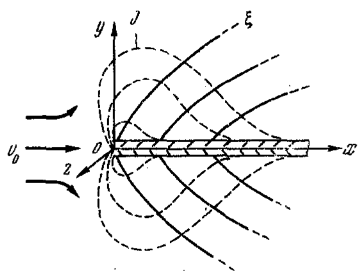

Fig. 2

$$\omega_k = ka^4 \frac{e}{mc} [(k^2 + k_x^2) H_0' + H_0'''] \alpha^*, \quad \alpha^* = [1 + (k^2 + k_x^2) a^2]^{-1}$$
$$\gamma_k = -(k^2 + k_x^2) a^2 \nu_f \alpha^*, \quad k_x^2 = \left(\frac{2\pi n}{\Delta x}\right)^2 \quad (n = 1, 2, 3, \ldots) \tag{2.6}$$

Hence we obtain a simple criterion for the instability of microscopic current vortices:

$$\nu_f = \frac{1}{mn} \frac{\partial R_f}{\partial v} < 0$$

This criterion indicates that the frictional force acting on an electron should decrease with increasing mean velocity. The force of Coulombic friction and also the force of collective friction can serve as examples of such forces in a plasma, if in it the mean thermal velocity v_{T_e} is replaced by the mean velocity v_e. The physical sense of this instability is that every small electron velocity perturbation that leads to a velocity increase decreases the frictional force and hence again leads to an increase in velocity perturbation.

The growth of microscopic vortical instabilities leads to an "anomalous-electron-viscosity" effect. It is easy to verify this, by averaging over the random-oscillation phases in the electron equation of motion. As a result of the averaging, in the total balance of forces for the electron component a new quantity Π_{xy} enters:

$$\Pi_{xy} = mn_0 \langle v_{x\sim} v_{y\sim}^* \rangle \quad (R_y = -\partial \Pi_{xy} / \partial x)$$

physically denoting the viscous-stress tensor, which results from the scattering of electrons by random vortical oscillations. In order to calculate this quantity we multiply Eq. (2.5) by the complex conjugate of the amplitude $H_\sim^*$, and we subtract the complex conjugate from the equality obtained. As a result, for the case in which $kd \ln H/dx \gg 1$, we obtain

$$\Pi_{xy} = -\sum_{\mathbf{k}} 2\pi m n_0 |v_{\mathbf{k}x}|^2 \delta(\omega - \mathbf{k}\mathbf{u}) \frac{\partial u}{\partial x},$$
$$v_{\mathbf{k}x} = -i \frac{kc}{4\pi e n} H_{\mathbf{k}} \tag{2.7}$$

The appearance of a δ function here indicates the resonance interaction mechanism of the sections of the electron-velocity profile $v_e(x)$ with excitable oscillations for which the phase velocity equals $\omega/k = v_e(x)$.

Equation (2.7) enables us to estimate the coefficient of anomolous electron viscosity

$$\eta = \sum_{\mathbf{k}} 2\pi n_0 m |v_{\mathbf{k}x}|^2 \delta(\omega - \mathbf{k}\mathbf{u})$$

This coefficient is proportional to the square of the vortical-oscillation amplitude $|v_{kx}|^2$ or the oscillation energy density. For this quantity, by analogy with ion-acoustic turbulence, we can write a kinetic equation. However, for the subsequent analysis it is not necessary that we do this; therefore we restrict ourselves only to the above qualitative representations.

3. We first consider the example of flow past a "magnetized plane electrode," represented by a magnetic dipole that is strongly stretched along the unperturbed-flow velocity v_0 formed, for example, by a system of linear conductors with a current (Fig. 2).

The boundary conditions in this problem are as follows:

$$H = H_0, \; v = 0 \;\text{ for }\; y = 0;$$
$$H \to H_\infty, \; n \to n_0, \; v \to v_0 \;\text{ as }\; y \to \pm\infty, \; x > 0 \tag{3.1}$$

In the ideal case the magnetized electrode can be assumed to be infinitesimally thin, so that the perturbations introduced by it into the plasma flow are small. In this case the initial system of equations can be linearized with respect to the small perturbations, and the nonlinearity can be taken into account only in terms of the frictional force R_f. The criterion of applicability of perturbation theory will involve the

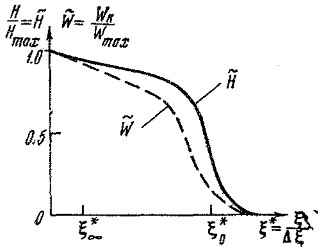

Fig. 3

smallness of the relative concentration fluctuations in the flow $(n-n_0)/n_0 \ll 1$. We represent the initial quantities in the form

$$V_x = v_0 + v_x,\ V_y = v_y,\ n = n_0 + n$$

Assuming that the magnetic field H, which is perpendicular to the flow velocity, is directed along the z axis, from the system of initial equations (1.2), (2.3), and (2.4) we obtain

$$\frac{\partial n}{\partial t} + v_0\frac{\partial n}{\partial x} + n_0\left(\frac{\partial v_x}{\partial x} + \frac{\partial v_y}{\partial y}\right) = 0$$

$$\frac{\partial v_x}{\partial t} + v_0\frac{\partial v_x}{\partial x} + \frac{H_0}{4\pi n_0 M}\frac{\partial H}{\partial x} + \frac{1}{n_0 M}\frac{\partial p_e}{\partial x} = 0$$

$$\frac{\partial v_y}{\partial t} + v_0\frac{\partial v_y}{\partial x} + \frac{H_0}{4\pi n_0 M}\frac{\partial H}{\partial y} + \frac{1}{n_0 M}\frac{\partial p_e}{\partial y} = 0$$

$$\frac{\partial H}{\partial t} + v_0\frac{\partial H}{\partial x} + H_0\left(\frac{\partial v_x}{\partial x} + \frac{\partial v_y}{\partial y}\right) - a^2\left(\frac{\partial}{\partial t} + v_0\frac{\partial}{\partial x}\right)\left(\frac{\partial^2 H}{\partial x^2} + \frac{\partial^2 H}{\partial y^2}\right) - \frac{\partial}{\partial x}\left(W_{k^*}\chi\frac{\partial H}{\partial x}\right) - \frac{\partial}{\partial y}\left(W_{k^*}\chi\frac{\partial H}{\partial y}\right) + \left(k_y^*\frac{\partial}{\partial x} - k_x^*\frac{\partial}{\partial y}\right)\frac{cW_{k^*}}{n_0 e} = 0$$

$$\left(\frac{\partial}{\partial t} + \frac{\partial}{\partial x}v_x + \frac{\partial}{\partial y}v_y\right) n \ln W_{k^*} = 2\left(\frac{c\mathbf{k}^*\,\mathrm{rot}\,\mathbf{H}}{4\pi e} - nv_s k^*\right) \tag{3.2}$$

Here $a = c/\omega_{pe}$ is the characteristic dispersion scale, which results from taking account of the electron inertia, the wave number k^*, and an arbitrary parameter $(k^* v_s \alpha^3/k_\parallel^* v_{T_e})$ affecting the rate of growth of microscopic turbulence, here and below assumed respectively equal to

$$k^* \approx \rho_{H_e}^{-1},\ k^* v_s a^3 / k_\parallel^* v_{T_e} \approx 1,\quad \chi = a^2 k^* / n_0 m v_s$$

The solution of the initial system of equations (3.2), which described the structure of the plasma flow in the magnetic boundary layer, can be conveniently investigated by the method of characteristics. We first find the expression, approximate in the framework of the linear theory, for the characteristics (we determine their slope with respect to the electrode plane), and then we analyze the small perturbations propagating along them.

Equations for the characteristics are obtained if in the initial system of equations (3.2) we neglect the collisionless dissipation and dispersion, i.e., if we set the noise energy density equal to zero and $a \to 0$:

$$\xi = y - x\,\mathrm{tg}\,\alpha = \mathrm{const},\quad \mathrm{tg}^2\alpha = \frac{H_0^2/4\pi n_0 M}{v_0^2 - H_0^2/4\pi n_0 M} \tag{3.3}$$

Here α is the slope of the corresponding characteristic with respect to the electrode plane. As a result of the nonlinearity in the slope of the characteristics with distance from the electrode, there is a decrease right down to zero in the unperturbed-flow region (Fig. 2); at the same time, the flow velocity along the characteristic montonically increases from zero to the unperturbed-flow velocity v_0, and the magnetic decreases from a maximum value H_0 at the electrode to zero in the unperturbed flow.

We consider the propagation of small perturbations along the characteristics. Introducing the coordinate of the characteristics $\xi = y - x\tan\alpha$ and making the following substitutions in the initial system of equations (3.2):

$$\frac{\partial}{\partial y} \to \frac{\partial}{\partial \xi},\quad \frac{\partial}{\partial x} \to -\mathrm{tg}\,\alpha\frac{\partial}{\partial \xi},\quad v_\xi = (v_y - v_x\,\mathrm{tg}\,\alpha)$$
$$-v_0\,\mathrm{tg}\,\alpha = v_{0\xi},\quad k_\xi^* = k_x^* + k_y^*\,\mathrm{tg}\,\alpha$$

we obtain equations that describe the propagation of small perturbations along the characteristics

$$\left(\frac{\partial}{\partial t} + v_{0\xi}\frac{\partial}{\partial \xi}\right)v_\xi + \frac{\partial}{\partial \xi}(1 + \mathrm{tg}^2\alpha)\left(\frac{H_0 H}{4\pi n_0 M} + \frac{p_e}{n_0 M}\right) = 0,\quad \frac{\partial n}{\partial t} + \frac{\partial}{\partial \xi}(n_0 v_\xi + n v_{0\xi}) = 0 \tag{3.4}$$

$$\frac{\partial H}{\partial t} + \frac{\partial}{\partial \xi}(v_{0\xi}H + H_0 v_\xi) - a^2(1 + \mathrm{tg}^2\alpha)\left(\frac{\partial}{\partial t} + v_{0\xi}\frac{\partial}{\partial \xi}\right)\frac{\partial^2 H}{\partial \xi^2} - \frac{\partial}{\partial \xi}\left[W_{k^*}\chi(1 + \mathrm{tg}^2\alpha)\frac{\partial H}{\partial \xi} - \frac{k_\xi^* c W_{k^*}}{e n_0}\right] = 0$$

$$\left(\frac{\partial}{\partial t} + v_{0\xi}\frac{\partial}{\partial \xi}\right)\ln W_k = \left(\frac{c k_\xi^*}{2\pi e n_0}\frac{\partial H}{\partial \xi} - 2k^* v_s\right)$$

The structure of these equations resembles the system of equations investigated by Ivanov and Rudakov [6], describing the dynamics of the quasilinear relaxation of a beam in a plasma. The role of the distribution function in the problem being considered is played by the magnetic field. However, before we apply the results of [6] to the system (3.4), we shall simplify it.

We first consider the general character of the process being described. The initial stage of perturbation growth on a plane plate, which generates in the half-space $x > 0$ an initial magnetic-field distribution $H_0(y)$, will proceed as follows. A rarefied plasma flow entering the region of unperturbed magnetic field begins to interact with it. In the plasma there arises an induction magnetic field maintained by a vortical current, which begins to displace the initial magnetic field. A nonlinear wave of compression of the magnetic field moves towards the electrode along a characteristic, where the wave profile, which is described by a simple Riemann wave, owing to the nonlinear distortion of the shape with time, increases in steepness. The "reversing" of such a wave is restrained by competing effects: dispersion and dissipation [7]. On the front of such a wave, as a result of a current instability there is excited a microscopic turbulence, the macroscopic effects of which qualitatively accurately describe the system of equations (3.4).

In conformity with [6] we investigate only the final stage of quasilinear relaxation, when the wave front is shaped in the form of a "steep step" (Fig. 3), where the plasma velocity in front of the front is close to zero, and behind the front it reaches its maximum value, close to the unperturbed-flow velocity v_0.

Neglecting the plasma velocity in front of the front in comparison with the velocity of the front itself, from the system of equations (3.4) we can obtain

$$\left(\frac{\partial}{\partial t} + v_{0\xi}\frac{\partial}{\partial \xi}\right)H = a^2(1+\mathrm{tg}^2\alpha)\left(\frac{\partial}{\partial t} + v_{0\xi}\frac{\partial}{\partial \xi}\right)\frac{\partial^2 H}{\partial \xi^2} + \frac{\partial}{\partial \xi} w_{k^*}\chi(1+\mathrm{tg}^2\alpha)\frac{\partial H}{\partial \xi}$$

$$\left(\frac{\partial}{\partial t} + v_{0\xi}\frac{\partial}{\partial \xi}\right)\ln\left(\frac{W_{k^*}}{W_0}\right) = \frac{ck_\xi^*}{2\pi e n_0}\frac{\partial H}{\partial \xi} \quad \text{for } j \gg j^* \tag{3.5}$$

These equations in structure agree with the similar equations investigated in [6] with accuracy up to the substitution

$$H \to f, \quad \frac{\partial}{\partial t} + v_{0\xi}\frac{\partial}{\partial \xi} \to \frac{\partial}{\partial t'}, \quad \frac{\partial}{\partial \xi} \to \frac{\partial}{\partial v} \quad \text{as} \quad a \to 0$$

In these equations, W_0 is the initial oscillation-energy density, equal to the thermal-fluctuation energy density, and

$$\left|\frac{ck_\xi^*}{2\pi e n_0 k^*}\frac{\partial H}{\partial \xi}\right| \gg v_s$$

The solution can be represented in the form

$$H = H_\infty\left[1 + \frac{\partial}{\partial \xi}\left(\frac{W_{k^*}(1+\mathrm{tg}^2\alpha)k^*}{(2n_0 M v_s\,\omega_{Hi,\infty} k_\xi^*)}\right)\right]$$

$$li\left(\frac{W_{k^*}}{W_0}\right) = \frac{1}{2}z_0(z_0 - z), \quad z = \frac{\xi}{(t-\xi/v_{0\xi})^{1/2}}\left[\frac{n_0 M v_s}{a^2 k^* W_0(1+\mathrm{tg}^2\alpha)}\right]^{1/2} \tag{3.6}$$

where z_0 is the initial coordinate, and li is a function of the integral logarithm.

An analysis of the solution shows that the magnetic-field front, represented by a steep step (Fig. 3), leads the noise front. This characteristic property of the dynamics of quasilinear relaxation, noted in [6], has a direct analogy with a "thermal wave," where the steepness of the front is due to the temperature dependence of the thermal conductivity being a power law. The equation for the thermal wave follows from (3.5) if we make the formal substitution $w_k^* \to T$, where T is the temperature in the thermal wave, and we set $a \equiv 0$.

To find a solution of (3.6), describing the leading part of the steep front, we neglected the dispersion effect, since it has no effect on the steepness of the leading front, which is completely determined by the growth rate of the noise energy. This can be verified in the following manner. We assume that at the leading front of the relaxation wave the dispersion effect balances the effect of the variation in steepness owing to the formation of a thermal wave; in other words, we will assume that at the leading front the force of electron inertia equalizes the force of turbulent friction. In this case we can expect that the steepness of the front will be flattened. We show, however, that this is not so.

Equating in system (3.5) the two types of forces and using the noise equation, after integration we find the following solution, which describes the leading front of the wave:

$$\frac{1}{W_{k^*}} = \frac{t'}{\rho_{H_e} n_0 m v_s} + \text{const}, \quad \frac{\partial H}{\partial \xi} = -\frac{2\pi e n_0 \chi}{c a^2 k_\xi^*} W_{k^*}, \quad k_\xi^* \approx \rho_{H_e}^{-1}$$

Hence we see that the characteristic dimension of the front

$$\Delta x = v_0 \Delta t' \approx \rho_{H_e} \frac{n_0 m v_s v_0}{W_{\max}} \approx \rho_{H_e} \left(\frac{v_s v_0}{u^2}\right)$$

strongly depends on the oscillation-energy density and can be made arbitrary small, for example, smaller than the dispersion scale $\Delta x \ll a$, by increasing the oscillation energy. An estimate was made above of the front width for the maximum energy density of ion-acoustic turbulence, which in order of magnitude equals $W_{\max} \approx 1/2\ nmu^2$.

Thus, the growth of microscopic turbulence favors the effect of an increase in the magnetic-field steepness and can lead to a reversal of the wave front and the generation of two-flow motion. However, even in the framework of the considered approximation there exists a restrictive mechanism. If the noises behind the wave front were to increase infinitely rapidly in comparison with the characteristic time of motion of the front, then the magnetic-field front would become infinitely steep and would reverse, but in actuality the growth time of the instability is finite; therefore, the dimensions of the front are finite and proportional to its velocity. If for given velocity of the front the microscopic turbulence is not able to ensure the magnetic pressure that is needed for this differential, then for such critical magnetic fields the wave front reverses, and two-flow motion appears.

Thus, the growth of ion-acoustic turbulence can lead to an increase in the steepness of the magnetic-field front and hence to a local gain of the current induced in the plasma. This effect of a "plasma micropinch" can also be connected with the special character of the dependence of the turbulent frictional force on the mean electron velocity $R_f(v_e)$, which decreases with increasing v_e. This condition is necessary for the excitation of microscopic vortical oscillations, whose growth can lead to the appearance of the effect of anomalous electron viscosity and, hence, to a mechanism stabilizing the effect of the increase of the steepness of the magnetic-field front. From the competition of the two opposing mechanisms we can find a stationary solution for the magnetic field and noise at the front. However, this investigation is beyond the scope of the present article.

We find the velocity of the front and the critical magnetic field. On the basis of the system of equations (3.4), we find the stationary solution $\partial/\partial t \equiv 0$, which describes the steady-state magnetic-field distribution along the characteristic. In this case the velocity of the front is approximately equal to the unperturbed-flow velocity:

$$v_f \approx v_0$$

These solutions are described by the equations

$$v_{0\xi}(H - H_\infty) = a^2(1 + \mathrm{tg}^2\,\alpha)\, v_{0\xi} \frac{\partial^2 H}{\partial \xi^2} + \chi W_{k^*}(1 + \mathrm{tg}^2\,\alpha) \frac{\partial H}{\partial \xi}$$
$$\frac{c k_\xi^*}{2\pi e n_0}(H - H_\infty) = v_{0\xi} \ln\left(\frac{W_{k^*}}{W_0}\right) \tag{3.7}$$

The solution for the magnetic field and the noise has the form

$$H = H_0\left[1 + \frac{v_{0\xi}}{2k_\xi^* a^2 \omega_{Hi,\infty}} \ln\left(\frac{W_{k^*}}{W_0}\right)\right]$$
$$\int_1^{W_{k^*}/W_0} dw \left\{\ln w \left[1 - \left(\frac{n_0 m v_s v_{0\xi}}{k^* a w}\right)^2 \frac{(1 - \ln w)}{(1 + \mathrm{tg}^2\,\alpha)}\right]\right\}^{-1} = \left(\frac{W_0 k^* a^2 M}{n_0 m v_s v_{0\xi} m}\right)\xi + \text{const} \tag{3.8}$$

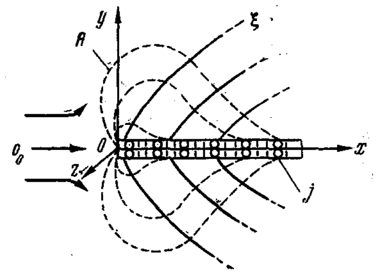

Fig. 4

It describes the leading part of the steep front and is applicable right up to values of the magnetic-field gradient

$$\partial H / \partial \xi \approx 2\pi e n_0 v_s k^* / c k_\xi^*$$

for the gently sloping part of the distribution, where the current velocity is close to the critical value. Assuming

$$\partial H / \partial \xi \approx H_\infty k^* v_s / 2 k_\xi^* a^2 \omega_{Hi,\infty}$$

in Eqs. (3.7), we find the velocity of the wave front $v_{0\xi}$ and the critical magnetic field H_c, beginning at which the wave reverses:

$$\begin{aligned} v_{0\xi} &= \left(\frac{H_\infty^2}{4\pi n_0 T_e}\right)^{1/2} \left[\frac{W_{\max}(1+\operatorname{tg}^2\alpha)}{m n_0 \ln(W_{\max}/W_0)}\right]^{1/2} \\ H_c &= H_\infty \left[1 + \frac{v_{0\xi}}{2k_\xi^* a^2 \omega_{Hi,\infty}} \ln\left(\frac{W_{\max}}{W_0}\right)\right] \end{aligned} \tag{3.9}$$

For the entire gently sloping part of the distribution (Figs. 2 and 3), with good accuracy the equality

$$\frac{dH}{d\xi} = \frac{2\pi e n_0 k^*}{c k_\xi^*} v_s \quad \text{or} \quad n T_e = \frac{c^2}{\pi \omega_{pi}^2} \left(\frac{k_\xi^2}{k^*}\right)^2 \left(\frac{dH}{d\xi}\right)^2 \tag{3.10}$$

is satisfied.

It can serve as the electron energy equation. Substituting this expression for the gas-kinetic pressure into (3.4), we obtain the following equation for the gently sloping part of the magnetic-field distribution:

$$\left(\frac{c}{\omega_{pi}} \frac{k_\xi^*}{k^*}\right)^2 \left(\frac{dH}{d\xi}\right)^2 = \left[n_0 M v_{0\xi}^2 \frac{(H - H_\infty)}{H_0(1+\operatorname{tg}^2\alpha)} - \frac{H_0(H - H_\infty)}{4\pi}\right] \tag{3.11}$$

We find the solution for $H \to H_\infty$, which "connects" with the steep part for $H \to H_0$:

$$2\left(\frac{c k_\xi^*}{\omega_{pi} k^*}\right) [(H - H_\infty) H_0]^{1/2} = \left[\frac{n_0 M v_{0\xi}^2}{(1+\operatorname{tg}^2\alpha)} - \frac{H_0^2}{4\pi}\right]^{1/2} (\xi - \xi_\infty) \tag{3.12}$$

Any width of the front, in order of magnitude, is equal to

$$\Delta\xi \approx 2\left(\frac{c k_\xi^*}{\omega_{pi} k^*}\right) \left[\frac{H_0 (H_0 - H_\infty)(1+\operatorname{tg}^2\alpha)}{n_0 M v_{0\xi}^2 - H_0^2 (1+\operatorname{tg}^2\alpha)/4\pi}\right]^{1/2} \approx \frac{c}{\omega_{pi}} \tag{3.13}$$

Thus, the solution for the boundary layer has been found. Such a boundary layer can be called collisionless since the mean free path of a particle can be considerably greater than its dimensions. Figures 2 and 3 show the structure of the boundary layer in this case. From the figures we see that near the electrode, where the steepness of the magnetic field is great, there is a large current directed opposite to the unperturbed-flow velocity. In this region there is strong damping of the principal flow. We write the balance for the forces acting on an ion in this region:

$$M n v_{ix} \frac{d v_{ix}}{dx} = - R_{fx} = - \frac{k^*}{2\omega_{k^*}} v_{ix} \frac{d W_{k^*}}{dx} \tag{3.14}$$

The ions are damped by the turbulent force of friction R_f. At the same time, the work going into the noise excitation and hence into the turbulent electron heating is equal to

$$(\mathbf{R}_f \mathbf{v}_i) \approx \frac{1}{24 (Mn)^3 v_s^3} \frac{d}{dx} W_{k^*}^3$$

4. We consider flow part past a magnetic dipole, which creates magnetic lines of force in the flow plane (Fig. 4). For definiteness we assume an infinitesimally thin dipole, so that the perturbations introduced by it into the flow are small. We consider the flow to be in the xy plane. The flow velocity has components v_x and v_y, the magnetic field correspondingly having the components H_x and H_y. We linearize the initial system of equations (1.2) and we take account of the nonlinearity only in terms of frictional force R_f:

$$\begin{aligned}
&\frac{\partial n}{\partial t}+v_0\frac{\partial n}{\partial x}+n_0\left(\frac{\partial v_x}{\partial x}+\frac{\partial v_y}{\partial y}\right)=0, \qquad \frac{\partial h_x}{\partial x}+\frac{\partial h_y}{\partial y}=0\\
&Mn_0\left(\frac{\partial v_x}{\partial t}+v_0\frac{\partial v_x}{\partial x}\right)=\frac{H_{0y}}{4\pi}\left(\frac{\partial h_x}{\partial y}-\frac{\partial h_y}{\partial x}\right)-\frac{\partial p_e}{\partial x}\\
&Mn_0\left(\frac{\partial v_y}{\partial t}+v_0\frac{\partial v_y}{\partial x}\right)=\frac{H_{0x}}{4\pi}\left(\frac{\partial h_y}{\partial x}-\frac{\partial h_x}{\partial y}\right)-\frac{\partial p_e}{\partial y}\\
&\left(\frac{\partial}{\partial t}+v_0\frac{\partial}{\partial x}\right)\ln\left(\frac{W_k}{W_0}\right)=\frac{ck^*}{2\pi en_0}\left(\frac{\partial h_y}{\partial x}-\frac{\partial h_x}{\partial y}\right)-2k^*v_s
\end{aligned}\tag{4.1}$$

$$\begin{aligned}
\frac{\partial h_x}{\partial t}&=\frac{\partial}{\partial y}\left\{v_xH_{0y}-v_yH_{0x}+v_0h_y+\frac{ckW_k}{en_0}\left[\frac{c}{4\pi en_0v_s}\left(\frac{\partial h_y}{\partial x}-\frac{\partial h_x}{\partial y}\right)-1\right]\right\}\\
\frac{\partial h_y}{\partial t}&=-\frac{\partial}{\partial x}\left\{v_xH_{0y}-v_yH_{0x}+v_0h_x+\frac{ck^*W_{k^*}}{en_0}\left[\frac{c}{4\pi en_0v_s}\left(\frac{\partial h_y}{\partial x}-\frac{\partial h_x}{\partial y}\right)-1\right]\right\}
\end{aligned}$$

We find the equations for the characteristics

$$\xi=y-x\operatorname{tg}\alpha,\quad \operatorname{tg}\alpha=\frac{(H_{0x}^2+H_{0y}^2)}{4\pi n_0Mv_0^2-(H_{0x}^2+H_{0y}^2)}\tag{4.2}$$

$$H_x=H_{0x}+h_x,\ H_y=H_{0y}+h_y,\ V_x=v_0+v_x,\ V_y=v_y$$

Here $a\to 0$, the current is directed along the z axis.

We consider stationary small perturbations along the characteristics. The system of equations that describes the distribution of magnetic field and of noise along the characteristic lines $\xi=y-x\operatorname{tg}\alpha$ has the form

$$\begin{aligned}
&(h_y-h_{y\infty})\left[1+\frac{(1+\operatorname{tg}^2\alpha)(H_{0x}^2+H_{0y}^2)}{\operatorname{tg}^2\alpha 4\pi Mn_0v_0^2}\right]=\frac{ck^*}{en_0v_0}W_{k^*}\left[\frac{c}{4\pi en_0v_s}\frac{(1+\operatorname{tg}^2\alpha)}{\operatorname{tg}\alpha}\frac{\partial H_y}{\partial\xi}\right]\\
&\ln\left(\frac{W_{k^*}}{W_0}\right)=\frac{ck^*}{2\pi en_0v_0}\frac{(1+\operatorname{tg}^2\alpha)}{\operatorname{tg}\alpha}(h_y-h_{y\infty})
\end{aligned}\tag{4.3}$$

Similarly we find the solution for the steep and gently sloping parts of the distribution of the magnetic-field front. For the steep part of the step we obtain the solution

$$\begin{aligned}
&h_y=h_{y\infty}\left[1+\frac{v_0}{2k^*a^2\omega_{Hi,\infty}}\ln\left(\frac{W_{k^*}}{W_0}\right)\right]\\
&li\left(\frac{W_{k^*}}{W_0}\right)=\frac{\operatorname{tg}\alpha n_0v_0v_s}{(1+\operatorname{tg}^2\alpha)W_0}\left[1+\frac{(1+\operatorname{tg}^2\alpha)(H_{0x}^2-H_{0y}^2)}{\operatorname{tg}^2\alpha\, 4\pi Mn_0v_0^2}\right]\frac{\xi}{a^2k^*}
\end{aligned}\tag{4.4}$$

This solution holds for all magnetic-field gradients right up to

$$\frac{\partial h_y}{\partial\xi}\gtrsim\frac{\operatorname{tg}\alpha}{(1+\operatorname{tg}^2\alpha)}\frac{4\pi en_0v_s}{c}$$

for which the current velocity is close to the critical value. For the gently sloping part, replacing the gas-kinetic pressure $p_e=nT_e$ by

$$p_e=\frac{(1+\operatorname{tg}^2\alpha)^2c^2}{4\pi\operatorname{tg}^2\alpha\omega_{pi}^2}\left(\frac{\partial H_y}{\partial\xi}\right)^2$$

we obtain the equation

$$\frac{(1+\operatorname{tg}^2\alpha)}{4\pi Mn_0v_0^2\operatorname{tg}^2\alpha}\frac{c^2}{\omega_{pi}^2}\left(\frac{\partial h_y}{\partial\xi}\right)^2=\frac{(h_y-h_{y\infty})}{(H_{0y}+H_{0x}/\operatorname{tg}\alpha)}\left[1-\frac{(H_{0x}^2+H_{0y}^2)(1+\operatorname{tg}^2\alpha)}{4\pi n_0Mv_0^2}\right]\tag{4.5}$$

It gives the solution

$$\frac{c}{\omega_{pi}}\left[\frac{(h_y-h_{y\infty})(H_{0y}+H_{0x}/\operatorname{tg}\alpha)(1+\operatorname{tg}^2\alpha)}{4\pi n_0Mv_0^2-(1+\operatorname{tg}^2\alpha)(H_{0x}^2+H_{0y}^2)}\right]^{1/2}=-\operatorname{tg}\alpha(\xi-\xi_\infty)\tag{4.6}$$

which "connects" for $\xi \to 0$ with the solution for the steep part. Substituting in Eq. (4.3) the critical value instead of the current velocity, we find the unperturbed-flow velocity v_0 and the critical magnetic field H_c:

$$v_0 = \frac{(H_{0x}^2 + H_{0y}^2)(1 + \mathrm{tg}^2\,\alpha)}{4\pi n T_e\,\mathrm{tg}\,\alpha}\left(\frac{W_{\max}}{n_0 m v_s \ln(W_{\max}/W_0)}\right)\left[1 + \frac{(1+\mathrm{tg}^2\,\alpha)(H_{0x}^2 + H_{0y}^2)}{\mathrm{tg}^2\,\alpha 4\pi n_0 M V_0^2}\right]^{-1}$$

$$H_c = H_{y\infty}\left[1 + \frac{v_0}{2k^* a^2 \omega_{Hi,\infty}} \ln\left(\frac{W_{\max}}{W_0}\right)\right] \tag{4.7}$$

Figure 4 shows the structure of the flow in this case. The dimension of the boundary layer in order of magnitude equals

$$\Delta\xi \approx c/\omega_{pi}$$

and can prove to be considerably less than the characteristic mean free path of a particle; therefore, this boundary layer can be called collisionless. The principal dissipation of energy and strong damping of the flow occur near the electrode in a narrow layer in which the maximum current density is localized. In this case there is turbulent ion damping. The energy separated during ion damping goes into turbulent electron heating.

Thus it has been shown that for the flow of a rarefied plasma stream past magnetized bodies, for a certain value of the flow velocity in the boundary layer a microscopic turbulence can be excited, leading to collisionless dissipation of the energy of the flow, turbulent plasma heating, and formation of a collisionless boundary layer, the dimension of which is less than the particle mean free path. However, for a certain relation between the flow velocity and the field intensity such a flow can prove to be unstable because of the reversing of the wave profile.

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